"More risk, more expected reward."

If you naively applied this central principle of financial theory to investing in equities over a 40-year career, you’d probably want to hold a portfolio consisting mostly of high-risk stocks. Given your long time horizon, the extra volatility most likely wouldn’t be a problem. You might start by buying a diversified collection of the most volatile stocks within an investable universe such as the Standard & Poor’s 500 Index. Once in a while, you’d look at your portfolio and find that some of your stocks had low volatility or had dropped outside the investable universe, so you’d replace them with newly available high-volatility stocks to stay true to your investment style. (Stock volatility here is measured on an absolute basis, not relative to a benchmark.)

To your horror, you would have significantly underperformed a cap-weighted index portfolio even within a decade. Adding insult to injury, holding the low-volatility stocks would have outperformed the index. What’s going on here?

The historical performance of different asset classes over the long term is indeed consistent with the principle that greater risk leads to greater reward. However, this principle does not extend to individual equities. Many academic studies, as well as historical backtests of low-volatility indexes, indicate that portfolios composed purely of low-volatility stocks tend to outperform portfolios of high-volatility stocks over the long term. This phenomenon, confirmed in studies of markets in many different countries, is usually interpreted to imply that low-volatility stocks have higher returns, on average. As this flies in the face of standard ideas about asset pricing, it is usually referred to as the “low-volatility anomaly.”

However, the average return of a portfolio is not simply the sum of the average return of the constituents; portfolio-level effects such as rebalancing and compounding play a role as well. It turns out that the anomalous performance of the above portfolios can actually be better understood by considering these effects. To continue to hold low-volatility stocks within a given investable universe with tolerable style drift, an investor must periodically sell stocks that have increased in volatility or fallen out of the universe. Applied diligently, this requirement becomes a rebalancing rule for portfolio construction. The difficulty in understanding what drives the performance of the resulting portfolio is that the rebalancing itself contributes to the portfolio return substantially.

The usual stock-based explanations try to relate the outperformance of portfolios of low volatility stocks to mispricings or systematic risk factors. In contrast, portfolio-based explanations allow quantification of the anomaly, suggest reasons for its long-term persistence and pave the way for more-efficient low-volatility portfolio construction. The key observation is that compounding has a significant impact on long-term returns. For example, during the past four years, many investors have learned the hard way that it takes a lot more than a 40 percent positive return to recover from a 40 percent downturn. This is because returns after the big downturn are compounding from a smaller base. There is no way to avoid this with an individual security in a buy-and-hold setting; the compound return really does reflect long-term performance.

The full impact of diversification and rebalancing on compound returns can be quantified using highly technical mathematics. Fortunately, the low-volatility anomaly can be analyzed using much simpler techniques. Let’s focus only on the impact on portfolio returns of the simple requirement that whenever we trade a portfolio, we are allowed to hold stocks only within the investable universe. Suppose that this universe consists of the 1,000 largest stocks (by capitalization) in the U.S. market. If we hold a stock that drops out of the universe, we will certainly be selling it at a relative loss, because the stock has failed to keep up with the other stocks. After all, it has lost its place in the top 1,000. In essence, the strategy requires that we sell low in this case, creating a drag on performance.

Of course, even a cap-weighted index of the same stocks is subject to this reconstitution drag. The question is, do portfolios of low-volatility stocks have less drag than the index? There is good reason to believe that they should and, furthermore, that portfolios of high-volatility stocks should have even more drag than the index. Everything follows from the basic observation that low-volatility stocks have less variability of outcomes, while high-volatility stocks have more variability. The effect on the reconstitution drag should be as follows:
Low-volatility stocks tend to move less in price than the average stock, so there is less risk of them dropping out of the universe than the average stock. On the other hand, high-volatility stocks have a higher risk of dropping out than the average stock.

When a low-volatility stock does drop out of the universe, it is likely to fall less than the average stock. The opposite is true for a high-volatility stock.

Low-volatility stocks are probably larger in capitalization, therefore less likely to drop out of the universe than the average stock. Conversely, high-volatility stocks are typically smaller in capitalization, therefore more likely to drop out of the universe than the average stock.

To get some sense of the effects of the reconstitution drag on performance, let’s imagine investing in a very simple strategy in which we buy and hold a single stock, replacing it only when it is no longer in the investable universe at the time of reconstitution. To simplify the situation even further, we’ll define this universe as consisting of stocks with capitalizations greater than $10 billion, and we will assume a monthly reconstitution. We start with one such stock. At the beginning of every subsequent month, there are two possibilities:

- The market capitalization of the company remains above $10 billion: continue to hold the stock.
- The company’s market capitalization has dropped below $10 billion: sell the stock and use the freed capital to buy the stock of the company whose market capitalization has most recently risen above $10 billion.

Not knowing anything further about the stocks involved, it is impossible to say what the short-term performance of this investment strategy will be. However, over many decades most large companies eventually falter for one reason or another. Whenever that happens for the stock being held, the strategy is to sell the stock at a loss and buy another stock after it has risen in value.

How much will this sell-low-buy-high trading hurt long-term portfolio performance? One way to think about it is to decompose the past monthly return of the stock into two pieces: the part that drove the capitalization down to $10 billion and the part that drove the capitalization even lower (see figure, above). It’s not fair to include the first piece in the cost analysis. Had the stock gone down so that the market capitalization was exactly $10 billion, we would have held on to it for another month, during which time it may have recovered. Instead, it is the excess negative return — labeled as “sell cost” in the figure — that really hurts. Moreover, the replacement stock was likely bought at a premium above the $10 billion threshold, contributing to the overall cost.

As a very long-term investment, this strategy is dubious at best: it can only work if the stock held never drops below $10 billion and will suffer greatly from reconstitution drag otherwise. The short-term performance will be poorest when the identity of the stock changes every month, as we will be losing money on every trade. If the stock we are holding is of high volatility, this is more likely to happen.
Of course, a single stock could go on for years, even decades, without dropping below the $10 billion threshold. We can speed up the process by investing in many stocks, which at the same time will allow us to cast new light on the persistent long-term underperformance of the megacap stocks with respect to broader stock indexes. Suppose we hold the largest 100 stocks at cap weight. Keeping the same monthly rebalancing schedule, we therefore sell the stocks that have dropped out of the top 100 at the end of the month and buy the stocks that have come into the top 100. This diligent rebalancing keeps us true to our megacap style, but it is also the sort of sell-low-buy-high trading that hurts the performance of the single-stock strategy. How much is the reconstitution drag in this case? For each stock that drops out of the universe, we can estimate the negative impact beyond the reconstitution boundary as follows:

\[
\frac{\text{capitalization of stock at the end of the month}}{\text{capitalization of 100th-largest stock at the end of the month}} - 1
\]

This quantity will be zero if the stock ends up exactly as the 100th largest; any greater drop makes it negative. This return should be multiplied by the capitalization weight of the stock at the beginning of the month and summed over all dropped stocks. However, this only accounts for the sell-low portion of the drag. The buy-high aspect is the result of buying into stocks that have recently outperformed and missing the chance to participate in that growth. Implementation and market structure details can affect the precise amount this premium contributes to the cost, but it can be reasonably approximated as equal to the sell-low cost. We arrive at an estimate of the total cost of rebalancing in the prescribed fashion by doubling the amount described above for the “sell-low” portion.

If our investment universe consists of the top 1,000 stocks, the above formula can be appropriately modified. In this case more stocks fall out (typically 15 to 30 per month compared to around 2 to 5 for the top 100), but the weights in these stocks are very small. Indeed, the total weight of these stocks exceeded 0.7 percent only during the crises of 2000 and 2008 (compared to between 0.5 and 2 percent for the top 100). As a result, we shouldn’t be surprised to see that the reconstitution drag for the top 1,000 stocks is smaller than that for the top 100 stocks. In fact, the drag on performance for the top 100 is 81 basis points a year greater than that for the top 1,000. This is comparable to the 85-basis-point-a-year average compound outperformance the top 1,000 stocks experience over the top 100.

Our first conclusion, and there is plenty of other evidence to support it, is that the long-term outperformance of the broader index over its concentrated megacap cousin has little if anything to do with individual stock return rates. Instead, that outperformance can be attributed to a consistently lower reconstitution drag. Of course, real stock indexes are reconstituted less frequently than monthly and often use rules that take more than just capitalization into account, but the general trend is still in favor of the broader universe. Small-cap stock indexes possess the additional advantage of having a reconstitution boost, as stocks that ascend out of the universe are sold at a profit and replaced with recent losers from the relevant large-cap index.

We can use the same techniques to analyze the reconstitution drag on portfolios of stocks selected purely by their volatility by considering the universe of the top 3,000 stocks. (There’s no reason we couldn’t stay with the top 1,000, but the effect we wish to analyze is stronger within the broader universe of stocks.) Volatility for any stock in the universe is estimated by analyzing the standard deviation of its daily returns (including dividends) for the previous year. Each of the following strategies is reconstituted monthly and is cap-weighted:

- Portfolio “LoVol” invests in the lowest 20 percent of the eligible names by volatility.
- Portfolio “HiVol” invests in the highest 20 percent of the eligible names by volatility.
- The index invests in all of the eligible names, cap-weighted.

Over the period from January 1974 through November 2011, LoVol beats the index by 59 basis points a year on a compound basis. Also, as predicted, the reconstitution drag on LoVol is indeed less than that of the index — by 57 basis points. As for HiVol, its performance relative to the index is truly dismal: it underperforms by 592 basis points a year on a compound basis. Again, as expected, the reconstitution drag on HiVol is greater than that for the index; the difference turns out to be a whopping 823 basis points a year, even though the total portfolio weight that contributes to it is typically about 1 to 5 percent per month. This 823-basis-point drag is greater than the 592-basis-point annual average underperformance, suggesting that the high-volatility stocks may actually have higher returns than the average stock on an individual basis. Unfortunately, the drag on performance associated with the trading required to stay true to style is so great that it overwhelms any such stock-level advantage.
In short, the low-volatility anomaly isn’t really an anomaly at all. Instead, this effect is a predictable consequence of the relationship between volatility and rebalancing costs, and it should not be surprising that it is observed in just about every data set available.

What would it take to make the effect disappear? Essentially, low-volatility stocks would have to start experiencing returns much lower than those of high-volatility stocks. This is highly unlikely to happen; after all, a similar arbitraging effect has not occurred with small-cap stocks, even though the small-minus-large size effect has been well known for decades. Moreover, even if a return differential arises in the future, the role of rebalancing will still be relevant.

Reducing portfolio volatility is a desirable objective in and of itself. While it is true that lower volatility does not automatically imply higher returns across asset classes (for example, a portfolio that invests 50 percent in a stock index and 50 percent in cash will have a lower expected return than a stock index portfolio), when one stays within the realm of fully invested equity portfolios, it appears that most legitimate efforts to lower portfolio volatility actually increase long-term returns. This is not as surprising as it might seem at first, because the cap-weighted index does not take full advantage of the compounding effect of rebalancing and is thus unlikely to be mean-variance-efficient. Additionally, systematically lowering portfolio volatility tends to result in adopting methodologies that are likely to reduce the reconstitution drag.

Investors can enjoy the best of both worlds: reduced volatility without lower return. Low-volatility equity portfolios offer a credible promise of long-term improvements in efficiency, exhibiting higher Sharpe ratios that are likely to persist independently of the existence of a high-volatility premium.

The above perspectives highlight the following pros and cons of some of the available categories of offerings in the low-volatility equity space:

**Portfolios of low-volatility stocks.** These are normally branded as low-volatility or defensive-style indexes. As such, they often have low-to-medium fees, are relatively transparently constructed and may well be more efficient than cap-weighted portfolios. However, their efficiency is unlikely to be optimal because of lack of control on both the risk and the reward sides.

**Minimum-variance portfolios.** These strategies should in principle profit from both rebalancing and optimization, and target portfolio volatility explicitly. Their fees are often low-to-medium as well. The methodology is not transparent: even the index-like offerings in this category usually depend on proprietary covariance estimation methodologies. Unconstrained minimization of variance tends to result in a huge turnover, so realistic implementations rely on stringent turnover controls, which may result in higher volatility than anticipated.

**Active low-volatility portfolios.** This class of strategies includes methodologies with an explicit alpha-generation mechanism independent of simply reducing volatility at the stock or portfolio level. Returns may be increased by traditional stock selection, fundamental overlays or quantitative factors. Because both risk and reward are carefully considered, portfolio efficiency may well be higher than in both of the above options. On the other hand, fees and turnover may be somewhat higher.

Regardless of the approach, low-volatility equity portfolios provide a way to gain efficiency over cap-weighted indexes. Understanding the construction and behavior of such strategies is significant for investors because volatility reduction allows greater allocation to equities at the same level of risk contribution. Rules-based approaches that reduce volatility will generally lead to more-efficient rebalancing. The efficiency can be further increased by targeting the portfolio-level rebalancing effect in a systematic manner. The nature and persistence of the low-volatility anomaly should not distract from the potential for low-volatility equity portfolios to be an extremely valuable component of an institutional investor’s overall portfolio.

*This material is for general informational purposes only. It is not intended as investment advice, as an offer or solicitation of an offer to sell or buy, or as an endorsement, recommendation, or sponsorship of any company, security, advisory service, or product. This information should not be used as the sole basis for investment decisions. Past performance is no guarantee of future results. Investing involves risk, including the loss of principal and fluctuation of value.*